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INVERSE PROBLEM OF FRACTURE MECHANICS FOR A DISK FITTED ONTO A ROTATING SHAFT

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A plane problem of fracture mechanics for a circular disk fitted onto a rotating shaft is considered. The disk is assumed to be fitted tightly onto the shaft, and there are N randomly located straight-line cracks of length $2l_k$ $(k = 1, 2, \ldots, N)$ near the inner surface of the disk. The interference between the disk and the rotating shaft, providing minimization of fracture parameters (stress intensity factor) of the disk, is theoretically studied on the basis of the minimax criterion. A closed system of algebraic equations is constructed, which allows the problem of optimal design to be solved. A simplified method of minimization of disk fracture parameters is considered.

Key words: *disk, rotating shaft, cracks, fitting interference, optimal design.*

Introduction. One of the main elements of the turbine is the disk fitted onto the shaft. Rotation induces inertial stresses in the disk, and the requirement of disk strength restricts the value of the admissible angular velocity of rotation. The operational life of the connection between the disk and the rotating shaft is determined by the stress distribution in zones of interaction of the elements of this connection. An important task at the current stage of engineering development is the optimal design of machine elements, providing better efficiency of their operation. The efficiency of the disk (rotor) fitted onto a rotating shaft can be controlled by changing structural design elements, in particular, the geometry (interference) of the connection between these elements. Solutions of such problems of mechanics were described in [1–9].

During operation of the disk (rotor) fitted onto the shaft, the fracture of the loaded disk occurs by the mechanism of formation of microcracks with which the disk continues to operate for a sufficiently long time. To make sure that the initial cracks with the most adverse locations will not grow to a critical size and will not cause fracture during the expected service life of the disk, it is necessary to perform the critical analysis of the disk.

Formulation of the Problem. Let us consider the stress–strain state of a circular disk fitted onto a shaft. The disk rotates together with the shaft with a constant angular velocity ω around the axis perpendicular to the shaft plane and passing through its center. Regimes of disk and shaft operation with possible residual strains are not allowed.

For the disk and the shaft, we use a polar coordinate system (r, θ) with the origin located at the center of concentric circles L and L_1 of radii R and R_1, respectively (see Fig. 1). It is assumed that the disk is fitted onto the shaft with interference (negative allowance). The interference function $g(\theta)$ is not known in advance and has to be determined from an additional condition. To simplify the problem, we assume that the disk and the shaft are subjected to plane strain conditions under the action of a self-balanced system of external normal and shear forces applied to the external circular boundary (contour L_1) of the disk. This allows us to neglect the spatial stress state in the shaft in the vicinity of the disk fitted with interference. It should be noted that the formulas derived in the present work under the assumption that the disk and the shaft have an identical length can be considered only as approximate formulas in the case with a short disk (rotor) fitted onto a much longer shaft. As the three-dimensional optimization problem is rather complicated, we confine our consideration to the plane approximation.

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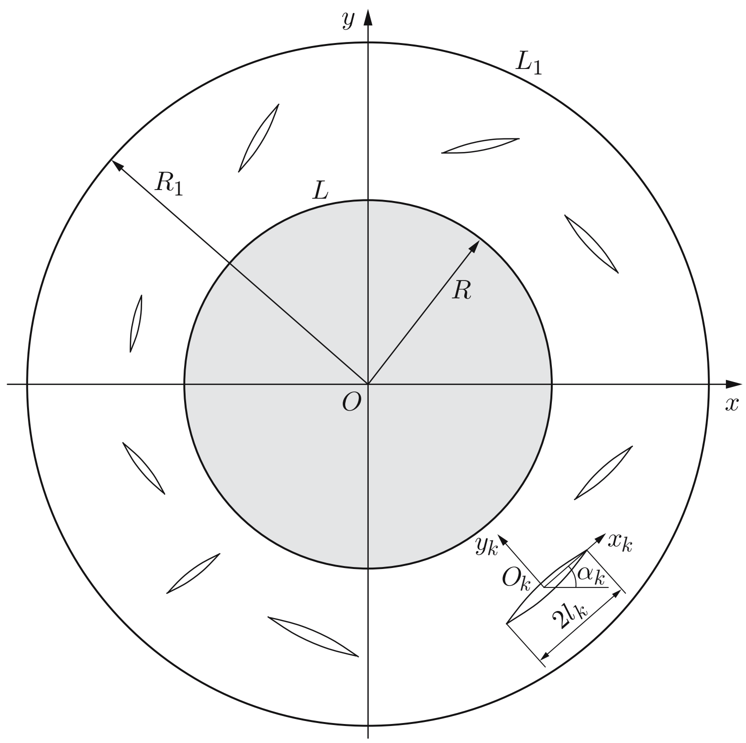

Fig. 1. Calculation scheme of the problem.

Let an elastic circular disk have N straight-line cracks of length $2l_k$ ($k = 1, 2, \ldots, N$). The crack centers are assumed to be the origins of the local coordinate systems $x_k O_k y_k$ with the $O_k x_k$ axes coinciding with the crack-propagation directions and forming angles α_k with the Ox axis (see Fig. 1). The crack faces are assumed to be free from external loads.

For the disk fitted onto the rotating shaft, the boundary conditions of the problem of the elasticity theory have the following form:

— at $r = R_1$,

$$\sigma_r = f_1(\theta), \qquad \tau_{r\theta} = f_2(\theta); \tag{1}$$

— at $r = R$,

$$\sigma_r - i\tau_{r\theta} = \sigma_r^b - i\tau_{r\theta}^b, \qquad u_2 - u_1 + i(v_2 - v_1) = g(\theta); \tag{2}$$

— on the crack faces,

$$\sigma_n = 0, \qquad \tau_{nt} = 0. \tag{3}$$

Here σ_r, σ_θ, $\tau_{r\theta}$, u_2, and v_2 are the stress and displacement components for the disk, σ_r^b, σ_θ^b, $\tau_{r\theta}^b$, u_1, and v_1 are the stress and displacement components for the shaft, $g(\theta)$ is the sought interference function, and $i = \sqrt{-1}$.

The sought complex function $g(\theta)$ characterizing the jumps of displacements due to the transition through the interface L between the media depends on the geometry of the connected elements before their deformation and on the method used to provide the contact of the points that belong to the cross-sectional contours of the shaft and the inner contour of the disk.

The problem formulation should be supplemented by a condition (criterion) that allows the interference function for the disk fitted onto the shaft to be determined.

According to the Irwin–Orowan theory of quasi-brittle fracture (see [10]), the parameter characterizing the stress state in the vicinity of the crack tip is the stress intensity factor. We can assume, therefore, that the disk fracture occurs at the moment when the stress intensity factor in the vicinity of the crack tip reaches the maximum value.

Studying the main fracture parameters and the influence of the disk–shaft fitting interference on these parameters, one can control the fracture by changing structural design elements, in particular, the fitting interference. As a criterion for determining the fitting interference [function $g(\theta)$], we use the condition of minimization of the maximum value of the stress intensity factor in the vicinity of the crack tip in the disk. Minimization of the maximum value of the stress intensity factor improves the efficiency of operation of the disk fitted onto the rotating shaft. We have to determine the fitting interference function $g(\theta)$ such that the stress field generated in the course of loading prevents the growth of cracks. Without loss of generality of the problem considered, we assume that the sought fitting interference function $g(\theta)$ can be presented as a Fourier series. Hence, the coefficients $A_k^{\rm int} = \alpha_k + i\beta_k$ of the expansion of the sought fitting interference functions should be chosen in a manner that ensures minimization of the maximum value of the stress intensity factor. This additional condition allows us to find the sought fitting interference function $g(\theta)$.

Case with One Crack. To solve the posed problem of optimal design, we have to solve the problem of fracture mechanics for the disk fitted onto the rotating shaft. In the case considered, we have equations of the plane elasticity theory in the presence of volume forces. The stress state in the rotating circular disk is presented as

$$\sigma_r = \sigma_r^0 + \sigma_r^1, \qquad \sigma_\theta = \sigma_\theta^0 + \sigma_\theta^1, \qquad \tau_{r\theta} = \tau_{r\theta}^0 + \tau_{r\theta}^1;$$

for the shaft, correspondingly, we obtain

$$\sigma_r^b = \sigma_r^{b0} + \sigma_r^{b1}, \qquad \sigma_\theta^b = \sigma_\theta^{b0} + \sigma_\theta^{b1}, \qquad \tau_{r\theta}^b = \tau_{r\theta}^{b0} + \tau_{r\theta}^{b1}.$$

Here the quantities σ_r^0, σ_θ^0, and $\tau_{r\theta}^0$ describe the stress state of the rotating disk, the quantities σ_r^{b0}, σ_θ^{b0}, and $\tau_{r\theta}^{b0}$ describe the stress state of the rotating shaft (the expressions for these stresses are known [11]), and σ_r^1, σ_θ^1, and $\tau_{r\theta}^1$ and σ_r^{b1}, σ_θ^{b1}, and $\tau_{r\theta}^{b1}$ are the components of the stresses induced by the fitting interference, cracks, and external loading.

The components of the displacement vectors for the disk and the shaft can be presented in a similar manner.

The boundary conditions for problem (1)–(3) can be written with the use of the Kolosov–Muskhelishvili formulas [11] in the form of a boundary-value problem for finding two pairs of complex potentials $\Phi(z)$ and $\Psi(z)$ for the disk and $\Phi_0(z)$ and $\Psi_0(z)$ for the shaft:

$$\Phi(\tau_1) + \overline{\Phi(\tau_1)} - [\bar{\tau}_1\Phi'(\tau_1) + \Psi(\tau_1)]\,e^{2i\theta} = f_1(\theta) - if_2(\theta),$$

$$\Phi(\tau) + \overline{\Phi(\tau)} - [\bar{\tau}\Phi'(\tau) + \Psi(\tau)]\,e^{2i\theta} = \Phi_0(\tau) + \overline{\Phi_0(\tau)} - [\bar{\tau}\Phi_0'(\tau) + \Psi_0(\tau)]\,e^{2i\theta}, \tag{4}$$

$$\varkappa\overline{\Phi(\tau)} - \Phi(\tau) + [\bar{\tau}\Phi'(\tau) + \Psi(\tau)]\,e^{2i\theta}$$

$$= (G/G_0)\{\varkappa_0\overline{\Phi_0(\tau)} - \Phi_0(\tau) + [\bar{\tau}\Phi_0'(\tau) + \Psi_0(\tau)]\,e^{2i\theta}\} + 2Gg'(\tau);$$

$$\Phi(t) + \overline{\Phi(t)} + t\overline{\Phi'(t)} + \overline{\Psi(t)} = -(\sigma_{y_1}^0 - i\tau_{x_1y_1}^0). \tag{5}$$

Here $\varkappa = 3 - 4\mu$, $\varkappa_0 = 3 - 4\mu_0$, μ and μ_0 are Poisson's ratios of the disk and shaft materials, respectively, G and G_0 are the shear moduli of the disk and shaft materials, respectively, $\tau_1 = R_1\exp(i\theta)$, $\tau = R\exp(i\theta)$, and t is the affix of the points of the crack faces.

The complex potentials $\Phi(z)$, $\Psi(z)$, $\Phi_0(z)$, and $\Psi_0(z)$ are sought in the form

$$\Phi(z) = \Phi_1(z) + \Phi_2(z) + \Phi_3(z), \qquad \Psi(z) = \Psi_1(z) + \Psi_2(z) + \Psi_3(z),$$

$$\Phi_1(z) = \sum_{k=-\infty}^{\infty} d_k z^k, \qquad \Psi_1(z) = \sum_{k=-\infty}^{\infty} c_k z^k,$$

$$\Phi_2(z) = \frac{1}{2\pi}\int_{-l_1}^{l_1} \frac{g_1(t)\,dt}{t - z_1}, \qquad \Psi_2(z) = \frac{1}{2\pi}\,e^{-2i\alpha_1}\int_{-l_1}^{l_1}\Big(\frac{\overline{g_1(t)}}{t - z_1} - \frac{\overline{T_1}\,e^{i\alpha_1}}{(t - z_1)^2}\,g_1(t)\Big)\,dt,$$

$$\Phi_3(z) = \frac{1}{2\pi}\int_{-l_1}^{l_1}\Big[\Big(-\frac{1}{z} - \frac{\overline{T_1}}{1 - z\overline{T_1}}\Big)\,e^{i\alpha_1}\,g_1(t) + e^{-i\alpha_1}\,\overline{g_1(t)}\,\frac{1 - T_1\overline{T_1}}{\overline{T_1}\,(1 - z\overline{T_1})^2}\Big]\,dt, \tag{6}$$

$$\Psi_3(z) = \frac{1}{2\pi z} \int_{-l_1}^{l_1} \Big[g_1(t)\, e^{i\alpha_1} \Big(\frac{1}{zT_1} - \frac{2}{z^2} - \frac{\overline{T_1}}{z(1 - z\overline{T_1}\,)} + \frac{\overline{T_1^2}}{(1 - z\overline{T_1}\,)^2} \Big)$$

$$+\, e^{-i\alpha_1}\, \overline{g_1(t)} \Big(\frac{1 - T_1\overline{T_1}}{z\overline{T_1}\,(1 - z\overline{T_1}\,)^2} - \frac{1}{1 - z\overline{T_1}} - \frac{2(1 - T_1\overline{T_1}\,)}{(1 - z\overline{T_1}\,)^3} \Big) \Big]\, dt;$$

$$\Phi_0(z) = \sum_{k=0}^{\infty} a_k z^k, \qquad \Psi_0(z) = \sum_{k=0}^{\infty} b_k z^k. \tag{7}$$

Here $T_1 = t\, e^{i\alpha_1} + z_1^0$, $z_1 = e^{-i\alpha_1}(z - z_1^0)$, and $g_1(x_1)$ is the sought function characterizing the discontinuity in displacements across the crack line:

$$g_1(x_1) = \frac{2G}{i(1+\varkappa)} \frac{\partial}{\partial x_1} [u_2^+(x_1, 0) - u_2^-(x_1, 0) + i(v_2^+(x_1, 0) - v_2^-(x_1, 0))].$$

Using Eqs. (6) and (7) for finding the complex potentials $\Phi_1(z)$, $\Psi_1(z)$, $\Phi_0(z)$, and $\Psi_0(z)$, we present the boundary conditions (4) in the form

$$\Phi_1(\tau_1) + \overline{\Phi_1(\tau_1)} - [\bar\tau_1 \Phi_1'(\tau_1) + \Psi_1(\tau_1)]\, e^{2i\theta} = f_1(\theta) - i f_2(\theta); \tag{8}$$

$$\Phi_1(\tau) + \overline{\Phi_1(\tau)} - [\bar\tau \Phi_1'(\tau) + \Psi_1(\tau)]\, e^{2i\theta} = \Phi_0(\tau) + \overline{\Phi_0(\tau)} - [\bar\tau \Phi_0'(\tau) + \Psi_0(\tau)]\, e^{2i\theta} - (f_3 - i f_4); \tag{9}$$

$$\varkappa\overline{\Phi_1(\tau)} - \Phi_1(\tau) + [\bar\tau \Phi_1'(\tau) + \Psi_1(\tau)]\, e^{2i\theta}$$

$$= (G/G_0)\{\varkappa_0 \overline{\Phi_0(\tau)} - \Phi_0(\tau) + [\bar\tau \Phi_0'(\tau) + \Psi_0(\tau)]\, e^{2i\theta}\} + 2Gg'(\tau) - (F_1 - iF_2). \tag{10}$$

Here

$$f_3 - i f_4 = \Phi_*(\tau) + \overline{\Phi_*(\tau)} - [\bar\tau \Phi_*'(\tau) + \Psi_*(\tau)]\, e^{2i\theta},$$

$$F_1 - iF_2 = \varkappa\overline{\Phi_*(\tau)} - \Phi_*(\tau) + [\bar\tau \Phi_*'(\tau) + \Psi_*(\tau)]\, e^{2i\theta},$$

$$\Phi_*(\tau) = \Phi_2(\tau) + \Phi_3(\tau), \qquad \Psi_*(\tau) = \Psi_2(\tau) + \Psi_3(\tau).$$

We denote the left side of the boundary condition (9) by $\sigma - i\tau$. Then, we obtain

$$\Phi_0(\tau) + \overline{\Phi_0(\tau)} - [\bar\tau \Phi_0'(\tau) + \Psi_0(\tau)]\, e^{2i\theta} - (f_3 - i f_4) = \sigma - i\tau. \tag{11}$$

We assume that the function $\sigma - i\tau$, which is a self-balanced system of forces acting on the shaft from the disk, is expanded into a complex Fourier series on the circular contour L [$\tau = R \exp(i\theta)$]:

$$\sigma - i\tau = \sum_{k=-\infty}^{\infty} A_k\, e^{ik\theta}.$$

We have condition (11) for determining the complex potentials $\Phi_0(z)$ and $\Psi_0(z)$ on the contour L. The functions $\Phi_0(z)$ and $\Psi_0(z)$ are analytical inside the shaft cross section $|z| \leqslant R$ and can be presented by series (7) (see [11]). Using the method of power series [11], we find the coefficients a_k and b_k of the potentials $\Phi_0(z)$ and $\Psi_0(z)$.

For determining the unknown quantities A_k, we consider the solution of the problem for the disk with $R_1 \leqslant |z| \leqslant R$. After some transformations, the complex potentials $\Phi_0(z)$ and $\Psi_0(z)$ make it possible to present the boundary conditions for finding the functions $\Phi_1(z)$ and $\Psi_1(z)$ in the form (8) and in the form

$$\Phi_1(\tau) + \overline{\Phi_1(\tau)} - [\bar\tau \Phi_1'(\tau) + \Psi_1(\tau)]\, e^{2i\theta} = \sum_{k=-\infty}^{\infty} A_k\, e^{ik\theta}; \tag{12}$$

$$\varkappa\overline{\Phi_1(\tau)} - \Phi_1(\tau) + [\bar\tau \Phi_1'(\tau) + \Psi_1(\tau)]\, e^{2i\theta} = \sum_{k=-\infty}^{\infty} A_k\, e^{ik\theta} + 2Gg'(\tau) - (F_1 - iF_2). \tag{13}$$

Here

$$A_0^* = \frac{G}{G_0}\,(\varkappa_0\bar{a}_0 - a_0 + b_2R^{-2}), \quad A_1^* = \frac{G}{G_0}\,(\varkappa_0\bar{a}_1R^{-1} + b_1R^{-1}), \quad A_2^* = \frac{G}{G_0}\,(\varkappa_0\bar{a}_2R^{-2} + b_0),$$

$$A_{-k}^* = \frac{G}{G_0}\,(b_{k+2}R^{-(k+2)} - a_k(1+k)R^{-k}), \qquad A_k^* = \frac{G}{G_0}\,\varkappa_0\bar{a}_kR^{-k}, \qquad k \geqslant 3.$$

We assume that the functions $f_1(\theta) - if_2(\theta)$, $g'(\tau)$, $f_3 - if_4$, and $F_1 - iF_2$ can be expanded into the Fourier series:

$$f_1 - if_2 = \sum_{k=-\infty}^{\infty} A_k'\,\mathrm{e}^{ik\theta}, \qquad g'(\tau) = \sum_{k=-\infty}^{\infty} A_k^{\mathrm{int}}\,\mathrm{e}^{ik\theta},$$

$$-(F_1 - iF_2) = \sum_{k=-\infty}^{\infty} D_k\,\mathrm{e}^{ik\theta}, \qquad f_3 - if_4 = \sum_{k=-\infty}^{\infty} B_k\,\mathrm{e}^{ik\theta}\,.$$

Here the coefficients D_k and B_k depend on the sought function $g_1(t)$ and are determined with the help of the theory of residues.

The boundary conditions (8), (12) allow us to determine the coefficients d_k and c_k, and the values of A_k can be found from the boundary condition (13). As a result, we obtain

$$d_0 = \frac{A_0'R_1^2 - A_0R^2}{2(R_1^2 - R^2)}, \quad d_{-1} = \frac{\overline{A}_1R}{1+\varkappa}, \quad c_{-1} = \varkappa\,\frac{A_1R}{1+\varkappa}, \quad d_1 = \frac{\overline{M}_{-1}}{R_1^4 - R^4} - \frac{2A_1R_1}{(1+\varkappa)(R_1^2+R^2)},$$

$$d_k = \frac{(1+k)(R_1^2 - R^2)M_k - \overline{M}_{-k}(R_1^{-2k+2} - R^{-2k+2})}{(1-k^2)(R_1^2 - R^2)^2 - (R_1^{2k+2} - R^{2k+2})(R_1^{-2k+2} - R^{-2k+2})} \qquad (k = \pm 2, \pm 3, \ldots),$$

$$c_{-2}R_1^{-2} = 2d_0 - A_0', \qquad c_{k-2}R^{k-2} = (1-k)d_kR^k + \bar{d}_{-k}R^{-k} - A_k,$$

$$M_k = A_k'R_1^{-k+2} - A_kR^{-k+2}, \qquad (1+k)\bar{d}_0 = A_0 + A_0^* + 2GA_0^{\mathrm{int}} + D_0,$$

$$(1+k)\bar{d}_kR^k = A_{-k} + A_{-k}^* + 2GA_{-k}^{\mathrm{int}} + D_{-k}, \qquad (1+k)\bar{d}_{-k}R^{-k} = A_k + A_k^* + 2GA_k^{\mathrm{int}} + D_k.$$

The right sides of the formulas for determining the coefficients a_k, b_k, c_k, d_k, and A_k include the coefficients of the expansions of the fitting interference function $g(\theta)$ and the integrals of the sought function $g_1(t)$.

We require that functions (6) satisfy the boundary conditions (4) on the crack faces and obtain a complex singular integral equation with respect to the unknown function $g_1(t)$:

$$\int_{-l_1}^{l_1} [R_{11}(t,x_1)g_1(t) + S_{11}(t,x_1)\overline{g_1(t)}\,]\,dt = \pi f_0(x_1), \qquad |x_1| \leqslant l_1,$$

$$f_0(x_1) = -\sigma_{y_1}^0 - [\Phi_1(x_1) + \overline{\Phi_1(x_1)} + x_1\overline{\Phi_1'(x_1)} + \overline{\Psi_1(x_1)}\,]. \tag{14}$$

Here we take into account that $\tau_{x_1y_1}^0 = 0$, the variables x_1, t, and l_1 are dimensionless quantities normalized to R, and the quantities R_{nk} and S_{nk} $(n = k = 1)$ are determined by formulas (VI.61) in [12].

The singular integral equation for the internal crack should be supplemented by the equality that describes the condition of uniqueness of displacements over the crack contour:

$$\int_{-l_1}^{l_1} g_1(t)\,dt = 0. \tag{15}$$

The singular integral equation (14) under condition (15) reduces to a system of M algebraic equations [12, 13] for M unknowns $g_1(t_m)$ $(m = 1, 2, \ldots, M)$:

$$\frac{1}{M}\sum_{m=1}^{M} l_1[g_1(t_m)R_{11}(l_1t_m, l_1x_r) + \overline{g_1(t_m)}\,S_{11}(l_1t_m, l_1x_r)] = f_0(x_r) \qquad (r = 1, 2, \ldots, M-1),$$

$$\sum_{m=1}^{M} g_1(t_m) = 0. \tag{16}$$

Here $t_m = \cos((2m-1)\pi/(2M))$ $(m = 1, 2, \ldots, M)$ and $x_r = \cos(\pi r/M)$ $(r = 1, 2, \ldots, M-1)$.

Passing to complex-conjugate quantities in Eq. (16), we obtain M additional algebraic equations. The obtained systems of equations with respect to a_k, b_k, d_k, c_k, A_k, and $g_1(t_m)$ $(m = 1, 2, \ldots, M)$ with a specified fitting interference $g(\theta)$ allow us to find the stress–strain state of the disk with a crack and, thus, to find the stress intensity factors.

The coefficients $A_k^{\rm int} = \alpha_k + i\beta_k$ $(k = 0, \pm 1, \pm 2, \ldots)$ have to be determined in the posed problem of optimal design. Therefore, the resultant algebraic system is not yet closed. We obtain the following relations for the stress intensity factors:

— in the vicinity of the right tip of the crack,

$$K_{\rm I} - iK_{\rm II} = \sqrt{\pi l_1}\sum_{m=1}^{M}(-1)^m g_1(t_m)\cot\Big(\frac{2m-1}{4M}\,\pi\Big); \tag{17}$$

— in the vicinity of the left tip of the crack,

$$K_{\rm I} - iK_{\rm II} = \sqrt{\pi l_1}\sum_{m=1}^{M}(-1)^{M+m} g_1(t_m)\tan\Big(\frac{2m-1}{4M}\,\pi\Big).$$

To construct the lacking equations, we minimize the maximum value of the stress intensity factor

$$K_{p,\max} \to \min$$

under the following constraints: absence of plastic deformations; condition of continuity of normal displacements in the disk and in the shaft $u_{n_1} = u_{n_2}$, which ensures the absence of discontinuities of the disk and shaft boundaries; condition $K_{p,\max} \leqslant K_{\rm th}$ ($K_{\rm th}$ is the characteristic of the threshold fracture toughness of the disk material, which is determined in experiments).

The optimization problem reduces to determining the coefficients (control parameters) of the expansion of the fitting interference function $g(\theta)$ into the Fourier series. The quantity $g_1(t_m)$ is a linear function of the coefficients $A_k^{\rm int}$ of the Fourier series of the fitting interference function $g(\theta)$. Therefore, the stress intensity factor (17) (objective function) is also a linear function of control parameters (control variables). Thus, the use of the minimax criterion reduces the optimization problem to a linear programming problem.

The problem for the case with nonuniform heating of the disk is considered in a similar manner.

The numerical calculations were performed by a simplex algorithm. The calculations were carried out for the disk made of ÉI417 steel and for the shaft made of St. 45 steel. The following problem parameters were used: $R_1 = 0.74$ m and $R = 0.03$ m; in the case with one crack, $\alpha_1 = 45°$ and $l_1/(R_1 - R) = 0.1$.

The load on the external contour of the disk was assumed to vary in accordance with the law $\sigma_r = p_0 \sin p_1\theta$, $\tau_{r\theta} = 0$, where p_0 and p_1 are specified parameters. The crack center was located at the point $z_1^0 = (R + a_1^0)\,{\rm e}^{i\pi\theta/18}$, where $a_1^0 = 0.2(R_1 - R)$.

Thus, we obtained the following values of the coefficients of the expansion of the function $g(\theta)$ into the Fourier series in the case of the optimal fitting interference: $\alpha_0 = 0.098$ mm, $\alpha_1 = 0.081$ mm, $\alpha_2 = 0.072$ mm, $\alpha_3 = 0.045$ mm, $\alpha_4 = 0.027$ mm, $\alpha_5 = 0.012$ mm, $\beta_0 = 0.076$ mm, $\beta_1 = 0.063$ mm, $\beta_2 = 0.057$ mm, $\beta_3 = 0.032$ mm, $\beta_4 = 0.025$ mm, and $\beta_5 = 0.009$ mm.

Case with an Arbitrary Number of Cracks. Let the elastic disk fitted onto the rotating shaft contain N straight-line cracks of length $2l_k$ $(k = 1, 2, \ldots, N)$ (see Fig. 1). Let us consider the problem of determining the fitting interference function for the disk fitted onto the rotating shaft, which ensures minimization of the maximum values of the stress intensity factor in the vicinities of the crack tips. The problem for this case is solved in a manner similar to solving the problem with one crack. The complex potentials $\Phi_2(z)$, $\Psi_2(z)$, $\Phi_3(z)$, and $\Psi_3(z)$ are generalized to the case of an arbitrary number of cracks. We require the boundary conditions on the crack faces to be satisfied and obtain a system of N singular integral equations with respect to the unknown functions $g_k(x_k)$ $(k = 1, 2, \ldots, N)$. The system of singular integral equations for internal cracks should be supplemented by the equalities

$$\int_{-l_k}^{l_k} g_k(t)\,dt = 0 \qquad (k = 1, 2, \ldots, N). \tag{18}$$

The system of singular integral equations under conditions (18) reduces to a system of $N \times M$ algebraic equations for determining $N \times M$ unknowns $g_n(t_m)$ $(n = 1, 2, \ldots, N;\ m = 1, 2, \ldots, M)$:

$$\frac{1}{M}\sum_{m=1}^{M}\sum_{k=1}^{N} l_k[g_k(t_m)R_{nk}(l_k t_m, l_n x_r) + \overline{g_k(t_m)}\,S_{nk}(l_k t_m, l_n x_r)] = f_n(x_r),$$

$$\sum_{m=1}^{M} g_n(t_m) = 0 \qquad (n = 1, 2, \ldots, N; \quad r = 1, 2, \ldots, M-1).$$

The following relations for the stress intensity factors are obtained:

— in the vicinity of the right crack tip $(n = 1, 2, \ldots, N)$,

$$K_{\mathrm{I}n} - iK_{\mathrm{II}n} = \sqrt{\pi l_n}\sum_{m=1}^{M}(-1)^m g_n(t_m)\cot\Big(\frac{2m-1}{4M}\,\pi\Big);$$

— in the vicinity of the left crack tip,

$$K_{\mathrm{I}n} - iK_{\mathrm{II}n} = \sqrt{\pi l_n}\sum_{m=1}^{M}(-1)^{M+m} g_n(t_m)\tan\Big(\frac{2m-1}{4M}\,\pi\Big).$$

With the use of the minimax criterion, the optimization problem considered in the case with an arbitrary number of cracks also reduces to a linear programming problem under the above-noted restrictions.

The optimal solution (coefficients α_k and β_k) allows the load-bearing capacity of the disk fitted onto the rotating shaft to be increased.

Simplified Method of Solving the Inverse Problem. If there are several cracks, the volume of calculations substantially increases. Let us consider a simplified method of solving the problem of determining the optimal fitting interference between the disk and the shaft.

In the expansion of the sought fitting interference function into the Fourier series, we retain only the number of terms coinciding with the number of crack tips. In the case with N internal cracks in the disk, we use $2N$ coefficients of the fitting interference function expansion into the Fourier series. We require that the stress intensity factors in the vicinities of the crack tips are equal to zero. Adding $2N$ linear algebraic equations to the main resolving equations (mentioned above), we obtain a closed algebraic system for determining all unknowns, including the coefficients α_k and β_k of the fitting interference function expansion into the Fourier series.

Let some part of the cracks N_1 reach the disk surface with one end. Then, the number of crack tips is $2N - N_1$. In the case where some of the cracks are surface cracks, we confine ourselves to $2N - N_1$ coefficients in the expansion of the sought fitting interference function into the Fourier series.

We require that the stress intensity factors in the vicinities of the crack tips are equal to zero. Adding these $2N - N_1$ linear algebraic equations to the main resolving equations, we again obtain a closed algebraic system for determining all unknowns.

It seems reasonable to use the simplified method of solving the problem of minimization of fracture parameters of the disk fitted onto the shaft in the case with a large number of cracks, where the amount of calculations by the simplex method becomes too large. In this case, these systems are solved numerically by the Gaussian method with the choice of the principal term. Therefore, the proposed methods of minimization of fracture parameters are mutually complementary.

Thus, the main resolving equations obtained in this work allow one to predict the growth of cracks in the disk by means of numerical calculations with determining the stress intensity factors for a given value of fitting interference between the disk and the shaft, to establish the admissible level of defects, and to calculate the maximum values of operation loads providing a sufficient safety margin. Solving the optimal design problem of determining the interference of fitting of the disk onto the shaft allows the optimal geometric parameters of the disk–shaft connection elements that improve the load-bearing capability to be chosen at the stage of design.

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